Stability of Shear Layers over Rough Surfaces

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ABSTRACT

Presence of surface imperfections may have significant impact on the stability of the adjacent shear layer and on the laminar-turbulent transition process in this layer. The main objective of the present study is the quantitative assessment of such effects in the case of distributed surface roughness. The analysis is focused on the linear stability theory and considers asymptotic as well as transient growth of flow perturbations. The explicit calculations are carried out in the context of channel flow.

1.0 INTRODUCTION

Understanding of the transition process in flows over rough surfaces is one of fundamental questions in fluid dynamics. This question is of practical interest in aerospace applications including the classical problem of the design of laminar airfoils as well as in the more current problem of the design of surface modifications for flow control. The latter application area involves development of strategies for improvement of mixing for better control/manipulation of the separations process in boundary layers and has significant potential for creation of means for active/passive manipulation of heat transfer in engines.

Flows over rough walls have been studied since the early works of Hagen [Hagen 1854] and Darcy [Darcy 1857], which were focused on turbulent regimes. Reynolds [Reynolds1883] was the first to pose the problem in the context of laminar-turbulent transition. In spite of many attempts [Schlichting 1979] the resolution of this problem is still uncertain. The original investigations involved measurements of flows in open channels and in pipes. Various possible roughness forms were classified using the concept of "equivalent roughness". Jimenez [Jimenez 2004] gives a nice description of the history and the current state-of-the-art of this concept. The phenomenological effects of the "equivalent roughness" are summarized in the form of friction coefficient [Nikuradse 1933], [Colebrook 1939], [Moody 1944]. These and other similar investigations show that surface roughness contributes directly to the dynamics of flow only if the wall is hydraulically rough. The concept of hydraulic smoothness is conceptually very appealing, however no precise criterion exists that would allow to predict if a given surface can be considered as being hydraulically smooth for flow conditions of interest. While the modelling concepts of this type have been continuously re-evaluated [Bradshaw 2000] and improved [Waigh 1998], they failed so far to uncover the mechanisms that govern the complex, flow-condition-dependent interaction between the roughness geometry and the moving fluid.

There were a number of concepts pursued in the attempt to link distributed surface roughness with the transition process. Singh [Sigh 1972], Lessen [Lessen 1976] and Kendall [Kendall 1981] considered roughness -distorted average velocity profile, but linear stability analysis of this profile proved inconclusive.

Merkle [Merkle 1977] considered concepts based on the roughness-induced additional mixing. Reshotko [Reshotko 1984] and Morkovin [Morkovin 1990] attempted to explain roughness effects in terms of Tollmien-Schlichting.

Floryan (1997) proposed a completely different approach where roughness geometry is modelled using spectral expansions; this formulation reduces the analysis of the effects of different geometries to scans of parameter space formed by the coefficients of the expansions. The problem is split into two steps. In the first one the basic state that accounts for the spatial structure of the roughness is determined numerically. In the second step, linear stability analysis that must account for the spatial structure of the roughness beyond a simple change of the average velocity profile is carried out. The above process is illustrated using the simplest possible geometry, i.e., channel with one wavy wall. Such wall geometry is represented in terms of a single Fourier mode and thus forms a very convenient reference case.

The laminar-turbulent transition occurs due to nonlinear effects induced by growing disturbances. In the case of asymptotic instability disturbances grow exponentially without any limits and always induce nonlinear effects. In the case of transient growth disturbances reach a certain magnitude but criterion that would permit to determine if this magnitude is sufficient to induce nonlinear effects is not known. Both growth mechanisms will be considered in this study.

This paper is organized as follows. Section 2 provides brief description of the basic state. Section 3 provides formulation of the asymptotic stability problem. Section 4 provides formulation of the transient growth problem. Section 5 provides discussion of selected results. Section 6 gives a short summary of the main conclusions.

2.0 BASIC STATE

The following presentation is limited to a short outline. Details are given by Floryan [Floryan1997]. Consider steady, two-dimensional flow in a channel with corrugated lower wall and smooth upper wall. The flow field is represented in the form

$$v_2(\mathbf{x}) = [u_2(x,y), v_2(x,y)] = \mathbf{V}_0(\mathbf{x}) + \mathbf{V}_1(\mathbf{x}) = [1-y^2, 0] + [u_1(x,y), v_1(x,y)],$$

$$p_2(\mathbf{x}) = p_0(\mathbf{x}) + p_1(\mathbf{x}) = -2x/Re + p_1(x),$$

$$(2.1)$$

where $\mathbf{V}_0$, p_0 describe the reference plane Poiseuille flow and $\mathbf{V}_1, p_1$ describe modifications associated with the presence of the corrugation. The channel extends from $-\infty$ to $+\infty$ in the x-direction and the locations of the lower and upper walls, i.e., $y_L(x)$ and $y_U(x)$, are defined as

$$y_L(x) = -1 + (Se^{i\alpha x} + CC), \quad y_U(x) = 1 , \tag{2.2}$$

where CC stand for the complex conjugate, and S and α denote the amplitude and the wave number of the wall corrugation, respectively. The flow is scaled using the maximum of the x-velocity of the reference Poiseuille flow and the average half-channel height h. The flow modifications can be represented using Fourier expansion in the form

$$\Psi(x,y) = \sum_{n=-\infty}^{n=+\infty} \Phi_n(y)e^{in\alpha x} \tag{2.3}$$

where Ψ represents the stream function defined in the usual manner, i.e., $u_1 = \partial_y \Psi$, $v_1 = -\partial_x \Psi$, $\Phi_n = \Phi_{-n}^*$ and star denotes complex conjugate. Elimination of pressure and use of (2.3) reduces the field equations to the equations for the functions Φ_n, $n \geq 0$, in the form

$$\left[D_n^2 - in\alpha Re\left(u_0 D_n - D^2 u_0\right)\right]\Phi_n - i\alpha Re \sum_{k=-\infty}^{k=+\infty}\left[kD\Phi_{n-k}D_k\Phi_k - (n-k)\Phi_{n-k}D_k D\Phi_k\right] = 0, \qquad (2.5)$$

where D=d/dy and $D_n = D^2 - n^2\alpha^2$. The boundary conditions at the channel walls are expressed in the following form

$$u_0\left(y_L(x)\right) + u_1\left(x, y_L(x)\right) = 0, \quad v_1\left(x, y_L(x)\right) = 0 , \qquad (2.6a)$$

$$u_1(x,l) = 0, \quad v_1(x,l) = 0 . \qquad (2.6b)$$

The required closing condition [Floryan 1997] has the form of the fixed mass flux constraint and can be expressed as

$$\Psi_0\left(y_U(x)\right) + \Psi\left(x, y_U(x)\right) = F + Q, \qquad (2.7a)$$

$$\Psi_0(-1) + \Psi(x,-1) = F, \qquad (2.7b)$$

where $\Psi_0 = -y^3/3 + y + 2/3$ denotes the stream function of the Poiseuille flow (continued analytically in $-1 < y < 1 + 2S$), Q stands for the (specified) volume flux and F denotes an arbitrary constant associated with introduction of the stream function (F denotes value of the stream function at the lower wall). In the present analysis F=0 and Q=4/3. The above problem is solved numerically with spectral accuracy using Chebyshev expansions. The boundary conditions at the bottom wall are implemented using the immersed boundary conditions concept [Szumbarski 1999][Floryan 2002] while the boundary conditions at the upper wall are implemented using a version of the tau technique.

3.0 ASYMPTOTIC LINEAR STABILITY PROBLEM

The analysis begins with the governing equations in the form of vorticity transport. Details of the formulation can be found in Floryan [Floryan 1997]. Unsteady, three-dimensional disturbances are superimposed on the mean part in the form

$$\boldsymbol{\omega} = \boldsymbol{\omega}_2(x, y) + \boldsymbol{\omega}_3(x, y, z, t), \quad \mathbf{v} = \mathbf{v}_2(x, y) + \mathbf{v}_3(x, y, z, t), \qquad (3.1)$$

where subscripts 2 and 3 refer to the mean flow and the disturbance field, respectively. The assumed form of the flow is substituted into the governing equations, the mean part is subtracted and the equations are linearized. The resulting linear disturbance equations have the form

$$\frac{\partial \boldsymbol{\omega}_3}{\partial t} + (\mathbf{v}_2 \cdot \nabla)\boldsymbol{\omega}_3 - (\boldsymbol{\omega}_3 \cdot \nabla)\mathbf{v}_2 + (\mathbf{v}_3 \cdot \nabla)\boldsymbol{\omega}_2 - (\boldsymbol{\omega}_2 \cdot \nabla)\mathbf{v}_3 = Re^{-1}\nabla^2 \boldsymbol{\omega}_3 , \qquad (3.2a)$$

$$\nabla \cdot \mathbf{v}_3 = 0 \quad , \quad \boldsymbol{\omega}_3 = \nabla \times \mathbf{v}_3 \qquad (3.2b,c)$$

and are subject to the homogeneous boundary conditions

$$\mathbf{v}_3(x,1,z) = 0, \quad \mathbf{v}_3(x, y_L(x), z) = 0 \tag{3.2d}$$

where y_L is given by Eq.(2.2). The mean flow has the form (Section 2)

$$\mathbf{v}_2(x,y) = [u_0(y), 0, 0] + \sum_{n=-\infty}^{n=\infty} \left[f_u^{(n)}(y), f_v^{(n)}(y), 0 \right] \exp(in\alpha x) \tag{3.3}$$

where $f_u^{(n)}, f_v^{(n)}$ represent solution to the problem (2.5)-(2.7) and $f_u^{(n)} = (f_u^{(-n)})^*$, $f_v^{(n)} = (f_v^{(-n)})^*$.

Partial differential equations (3.2) have coefficients that are functions of y and x, being periodic in x. The t and z dependence can be separated in the usual manner leading to the solution in the form

$$\mathbf{v}_3(x,y,z,t) = \sum_{m=-\infty}^{m=+\infty} \left[g_u^{(m)}(y), g_v^{(m)}(y), g_w^{(m)}(y) \right] e^{i[(\delta+m\alpha)x + \beta z - \sigma t]} + CC \tag{3.4}$$

where δ and β are real and account for the streamwise and spanwise periodicity of the disturbance field, respectively. The exponent σ is complex and its imaginary and real parts describe the rate of growth and the frequency of the disturbances, respectively.

Substitution of (3.3) and (3.4) into the disturbance equations (3.2) and separation of Fourier components results in a system of ordinary differential equations governing $g_u^{(m)}, g_v^{(m)}, g_w^{(m)}$. It is convenient for analysis purposes to express these relations in terms of the y-components of disturbance vorticity $\theta^{(m)} = t_m g_w^{(m)} - \beta g_u^{(m)}$ and disturbance velocity $g_v^{(m)}$, i.e.,

$$S^{(m)}\theta^{(m)} + Cg_v^{(m)} = \text{Re} \sum_{n=-\infty}^{n=\infty} (E_v^{(m,n)} g_v^{(m-n)} + E_\theta^{(m,n)} \theta^{(m-n)}), \tag{3.5a}$$

$$T^{(m)}g_v^{(m)} = -\text{Re} \sum_{n=-\infty}^{n=\infty} (H_v^{(m,n)} g_v^{(m-n)} + H_\theta^{(m,n)} \theta^{(m-n)}), \qquad -\infty < m < \infty \tag{3.5b}$$

where the explicit forms of the operators E, H are given in the Appendix A. The boundary conditions have the form

$$g_u^{(m)}(1) = g_v^{(m)}(1) = g_w^{(m)}(1) = g_u^{(m)}(y_L) = g_v^{(m)}(y_L) = g_w^{(m)}(y_L) = 0, \quad -\infty < m < +\infty. \tag{3.6}$$

Conditions at the upper wall are imposed using a variation of the tau technique, while those at the bottom wall are implemented using immersed boundary conditions concept [Floryan 2002]. Equations (3.5) with boundary conditions (3.6) have nontrivial solutions only for certain combinations of parameters δ, σ and β. The required dispersion relation has to be determined numerically. For the purposes of calculations, the problem is posed as an eigenvalue problem for σ.

4.0 LINEAR TRANSIENT GROWTH PROBLEM

The analysis begins with the same formulation as used in the previous section. The following presentation is limited to a short outline. Details are given by Szumbarski [Szumbarski 2004]. The disturbances are assumed in the form

$$\mathbf{v}_3(t,x,y,z) = [\hat{u}(x,y,t),\hat{v}(x,y,t),\hat{w}(x,y,t)]e^{i\delta x+i\beta z} = \sum_{m=-\infty}^{m=+\infty}\left[g_u^{(m)}(t,y),g_v^{(m)}(t,y),g_w^{(m)}(t,y)\right]e^{i(\gamma_m x+\beta z)} + CC \quad (4.1)$$

where β and δ are real and denote spanwise and streamwise wave numbers, respectively, $\gamma_m = \delta + m\alpha$, $\hat{u},\hat{v},\hat{w}$ describe modulation of disturbances by the corrugation and are periodic in x. Equations (4.1) and (3.3) are substituted into (3.2), the unknowns are expressed in terms of the normal velocity component v_3 and the normal vorticity component ω_{Dy}, where

$$\omega_{Dy} = \frac{\partial u_3}{\partial z} - \frac{\partial w_3}{\partial x} = -i\sum_{m=-\infty}^{m=+\infty}\theta^{(m)}(t,y)\,e^{i(\gamma_m x+\beta z)}, \quad \theta^{(m)} = -\beta g_u^{(m)} + \gamma_m g_w^{(m)} \quad (4.2)$$

and the Fourier modes are separated resulting in the following form of disturbances equations for $\theta^{(m)}, g_v^{(m)}$, $m \in (-\infty, +\infty)$

$$\left(D^2 - k_m^2\right)\partial_t g_v^{(m)} - Re^{-1}\left\{\left(D^2 - k_m^2\right)^2 - iRe\gamma_m\left[u_0\left(D^2 - k_m^2\right) - D^2 u_0\right]\right\}g_v^{(m)} = \quad (4.3a)$$

$$= Re^{-1}\sum_{n=1}^{\infty}\left(\hat{G}_v^{(m,n)}g_v^{(m+n)} + G_v^{(m,n)}g_v^{(m-n)} + \hat{G}_\theta^{(m,n)}\theta^{(m+n)} + G_\theta^{(m,n)}\theta^{(m-n)}\right) + G_v^{(0)}g_v^{(m)}$$

$$\partial_t\theta^{(m)} - Re^{-1}\left[D^2 - k_m^2 - iRe\gamma_m u_0\right]\theta^{(m)} + \beta Du_0\,g_v^{(m)} = \quad (4.3b)$$

$$= Re^{-1}\sum_{n=1}^{\infty}\left(\hat{S}_v^{(m,n)}g_v^{(m+n)} + S_v^{(m,n)}g_v^{(m-n)} + \hat{S}_\theta^{(m,n)}\theta^{(m+n)} + S_\theta^{(m,n)}\theta^{(m-n)}\right) + S_v^{(0)}g_v^{(m)} + S_\theta^{(0)}\theta^{(m)} \quad .$$

The explicit forms of the operators $G_v, G_\theta, \hat{G}_v, \hat{G}_\theta, S_v, S_\theta, \hat{S}_v, \hat{S}_\theta$ are given in the Appendix B. It can be shown that boundary conditions (3.3) take the form

$$g_u^{(m)}(t,y_L(x)) = 0, g_v^{(m)}(t,y_L(x)) = 0, g_w^{(m)}(t,y_L(x)) = 0, \theta^{(m)}(t,y_L(x)) = 0 \quad (4.4a)$$

$$g_u^{(m)}(t,1) = 0, g_v^{(m)}(t,1) = 0, g_w^{(m)}(t,1) = 0, \theta^{(m)}(t,1) = 0 \quad . \quad (4.4b)$$

where $m \in (-\infty, +\infty)$. The boundary conditions at the upper wall are implemented using a version of the tau technique, while boundary conditions at the lower wall are implemented using the immersed boundary conditions concept. Problem (4.3)-(4.4) has to be supplemented by the initial conditions. The solution involves search for the form of the initial conditions that gives the largest possible transient growth (optimal disturbances).

5.0   DISCUSSION OF RESULTS

The unstable disturbances have the form of either streamwise vortices or travelling waves [Floryan 2003]. The vortices are induced by the presence of the roughness, but only when the roughness amplitude and wave number are in the correct range. Figure 1 displays a typical stability diagram for the streamwise vortices. Reduction of either the roughness amplitude S or the flow Reynolds number Re reduces the unstable zone. Critical conditions are found when this zone shrinks to a single point.
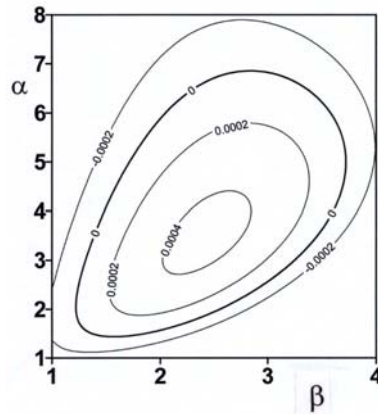

Figure 1.   Amplification rates Imag(σ) of disturbances in the form of streamwise vortices as a function of the roughness wave number α and the vortex wave number β for the flow Reynolds number Re=5000 and the roughness amplitude S=0.0075.

Travelling waves become unstable when the Reynolds number reaches the critical value of Re=5772 in a smooth channel. Presence of roughness significantly reduces this value. Figure 2 displays a typical stability diagram in the case of rough channel. The size of the unstable zone decreases when either the roughness amplitude S or the flow Reynolds number Re decrease. The critical conditions are found when this zobe shrinks to zero.
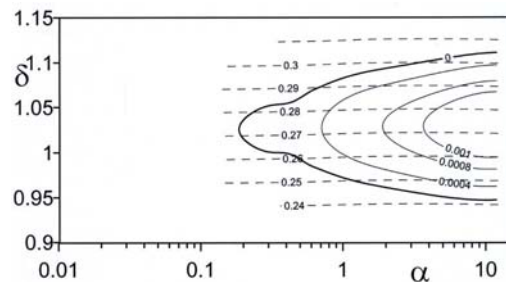

Figure 2.   Curves of constant amplification Imag(σ) for disturbances in the form of two-dimensional travelling waves for the roughness amplitude S=0.0085 and the flow Reynolds number Re=5600 as a function of the roughness wave number α and the disturbance wave number δ. Dash lines correspond to Real(σ).

Figure 3 displays critical curves for both types of instability. The area below these curves defines conditions when the wall behaves as hydraulically smooth. There will be at least one roughness wave number that would induce instability for a given roughness amplitude when Reynolds number increases above the critical value defined by these curves.
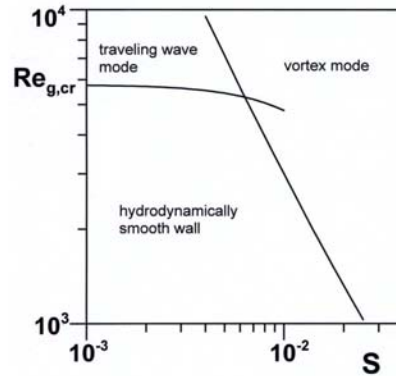
Figure 3. Variations of the global critical Reynolds number Re$_{g,cr}$ describing the travelling wave instability and the vortex-like instability as a function of the roughness amplitude S. The area below and to the left of both curves corresponds to the flow conditions and the wall geometry that do not produce any instability. Such wall operating under the specified Reynolds number behaves as the hydraulically smooth wall.

The definition of the hydraulically smooth wall stated above is based on the asymptotic instability. Such instability always induces transition. The reader should note that this definition omits transient growth that may induce transition under subcritical conditions. The formulation presented in Section 4 provides means for assessing the strength of the transient growth. This issue is currently subject to detailed investigation.

6.0 SUMMARY

Formulation of the problem of growth of linear disturbances in a rough channel is presented. The roughness geometry is represented in terms of Fourier series. Two types of disturbance growth are considered, i.e., exponential growth associated with the asymptotic instability and transient growth. Detailed results are given in the case of roughness described by a single Fourier mode and asymptotic instability. Conditions under which roughness is not hydraulically active have been identified using criterion based on the roughness ability to induce asymptotic instability.

7.0 APPENDIX A

Operators used in Eqs. (3.6) have the following definitions:

$$E_v^{(m,n)} = \beta(-Df_u^{(n)} + in\alpha k_{m-n}^{-2} f_v^{(n)} D^2); \quad E_\theta^{(m,n)} = it_m f_u^{(n)} + (1 + n\alpha t_{m-n} k_{m-n}^{-2}) f_v^{(n)} D ;$$

$$H_v^{(m,n)} = in\alpha k_{m-n}^{-2}(\beta^2 - t_m t_{m-n})Df_u^{(n)}D + k_m^2 k_{m-n}^{-2}(\beta^2 + t_{m-n} t_{m-2n}) f_v^{(n)}D +$$
$$ik_{m-n}^{-2}(-k_{m-n}^2 t_m + 2n\alpha\beta) f_u^{(n)}D^2 + k_{m-n}^{-2}(n\alpha t_m - k_m^2) f_v^{(n)}D^3 + \qquad ;$$
$$ik_m^2 t_{m-2n} f_u^{(n)} + it_m D^2 f_u^{(n)}$$

$$H_\theta^{(m,n)} = n\alpha\beta(2t_{m-n} k_{m-n}^{-2} f_u^{(n)}D + k_{m-n}^{-2}(t_m + t_{m-n})Df_u^{(n)} - ik_m^2 k_{m-n}^{-2} f_v^{(n)} - ik_{m-n}^{-2} f_v^{(n)}D^2) ;$$

where D=d/dy, t$_m$=mα+δ and $k_m^2 = t_m^2 + \beta^2$.

8.0   APPENDIX B

Operators used in Eqs (4.3) are defined as follows:

$$G_v^{(m,n)} = Re\left[\frac{i\,n\alpha}{k_{m-n}^2}\left(\beta^2 - \gamma_m\gamma_{m-n}\right)Df_u^{(n)}\,D + \frac{k_m^2}{k_{m-n}^2}\left(\beta^2 + \gamma_{m-n}\gamma_{m-2n}\right)f_v^{(n)}\,D + \right.$$

$$\left. + \frac{i}{k_{m-n}^2}\left(2n\alpha\beta^2 - \gamma_m k_{m-n}^2\right)f_u^{(n)}\,D^2 + \frac{i}{k_{m-n}^2}\left(n\alpha\gamma_m - k_m^2\right)f_v^{(n)}\,D^3 + i\,k_m^2\gamma_{m-2n}f_u^{(n)} + i\,\gamma_m D^2 f_u^{(n)} \right] \quad ;$$

$$\hat{G}_v^{(m,n)} = Re\left[\frac{i\,n\alpha}{k_{m+n}^2}\left(\gamma_m\gamma_{m+n} - \beta^2\right)\left(Df_u^{(n)}\right)^*D + \frac{k_m^2}{k_{m+n}^2}\left(\beta^2 + \gamma_{m+n}\gamma_{m+2n}\right)\left(f_v^{(n)}\right)^*D + \right.$$

$$\left. + \frac{i}{k_{m+n}^2}\left(-2n\alpha\beta^2 - \gamma_m k_{m+n}^2\right)\left(f_u^{(n)}\right)^*D^2 + \frac{i}{k_{m+n}^2}\left(-n\alpha\gamma_m - k_m^2\right)\left(f_v^{(n)}\right)^*D^3 + + i\,k_m^2\gamma_{m+2n}\left(f_u^{(n)}\right)^* + i\,\gamma_m\left(D^2 f_u^{(n)}\right)^* \right] \quad ;$$

$$G_v^{(0)} = -i\gamma_m[f_u^{(0)}(D^2 - k_m^{(2)}) - D^2 f_u^{(0)}] \quad ;$$

$$G_\theta^{(m,n)} = Re\left[\frac{1}{k_{m-n}^2}2n\alpha\beta\gamma_{m-n}f_u^{(n)}\,D + \frac{n\alpha\beta}{k_{m-n}^2}\left(\gamma_m + \gamma_{m-n}\right)Df_u^{(n)} - \frac{in\alpha\beta k_m^2}{k_{m-n}^2}f_v^{(n)} - \frac{in\alpha\beta}{k_{m-n}^2}f_v^{(n)}\,D^2 \right];$$

$$\hat{G}_\theta^{(m,n)} = Re\left[-\frac{1}{k_{m+n}^2}2n\alpha\beta\gamma_{m+n}\left(f_u^{(n)}\right)^*D - \frac{n\alpha\beta}{k_{m+n}^2}\left(\gamma_m + \gamma_{m+n}\right)\left(Df_u^{(n)}\right)^* + + \frac{in\alpha\beta k_m^2}{k_{m+n}^2}\left(f_v^{(n)}\right)^* + \frac{in\alpha\beta}{k_{m+n}^2}\left(f_v^{(n)}\right)^*D^2 \right] \quad ;$$

$$S_v^{(m,n)} = Re\left[\beta\,Df_u^{(n)} - \frac{in\alpha\beta}{k_{m-n}^2}f_v^{(n)}\,D^2 \right] \quad ; \qquad \hat{S}_v^{(m,n)} = Re\left[\beta\left(Df_u^{(n)}\right)^* + \frac{in\alpha\beta}{k_{m+n}^2}\left(f_v^{(n)}\right)^*D^2 \right] \quad ;$$

$$S_v^{(0)} = -\beta Df_u^{(0)} \quad ; \quad S_\theta^{(m,n)} = Re\left[-i\gamma_m f_u^{(m)} - \frac{1}{k_{m-n}^2}\left(\beta^2 + \gamma_m\gamma_{m-n}\right)f_v^{(n)}\,D \right] \quad ;$$

$$\hat{S}_\theta^{(m,n)} = Re\left[-i\gamma_m\left(f_u^{(m)}\right)^* - \frac{1}{k_{m+n}^2}\left(\beta^2 + \gamma_m\gamma_{m+n}\right)\left(f_v^{(n)}\right)^*D \right] \quad ; S_\theta^{(0)} = -i\gamma_m f_u^{(0)} \quad .$$

9.0   REFERENCES

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